Released Items
Mathematics 30–1

Diploma Examinations Program 2019

This document was written primarily for:

Students	✓	
Teachers	✓	of Mathematics 30–1
Administrators	✓	
Parents		
General Audience		
Others		

Alberta Education, Government of Alberta

2019–2020

Mathematics 30–1 Released Items

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Please note that if you cannot access one of the direct website links referred to in this document, you can find diploma examination-related materials on the Alberta Education website.

Introduction

The questions in this booklet are from the *January 2019 Form 1 Mathematics 30-1 Diploma Examination*. Teachers may wish to use these questions in a variety of ways to help students develop and demonstrate an understanding of the concepts described in the Mathematics 30-1 Program of Studies. This material, along with the Program of Studies, Information Bulletin, and Assessment Standards and Exemplars, can provide insights that assist with decisions about instructional planning.

These questions are released in both English and French by the Provincial Assessment Sector.

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Additional documents

The Provincial Assessment Sector supports the instruction of Mathematics 30–1 with the following documents available online.

- *Mathematics 30-1 Information Bulletin*
- *Mathematics 30-1 Assessment Standards and Exemplars*
- *Mathematics 30-1 Released Materials*
- *Mathematics 30-1 Written-Response Information*
- *School Reports and Instructional Group Reports* (Detailed statistical information is provided on provincial, group, and individual student performance on January and June diploma examinations.)

Mathematics 30–1 Diploma Examination January 2019 Form 1 – Item Information

The following tables give the results for the machine-scored and written-response questions released from the examination. For each question, the table also gives the correct response, the topic, the outcome, the cognitive level, and the assessment standard.

Topics		**Cognitive Levels**	**Standards**
RF	Relations and Functions	Conceptual	Acceptable
TRIG	Trigonometry	Procedural	Excellence
PCBT	Permutations, Combinations, and Binomial Theorem	Problem Solving	

Question	Diff.*	Key	Topic	Outcome	Cognitive Level	Standard
MC1	83.7%	B	RF	2	Procedural	Acceptable
MC2	86.3%	D	RF	3, 5	Conceptual	Acceptable
MC3	83.8%	D	RF	3	Conceptual	Acceptable
MC4	84.9%	C	RF	4	Procedural	Excellence
MC5	66.0%	B	RF	5	Conceptual	Acceptable
MC6	53.5%	C	RF	6	Conceptual	Excellence
MC7	77.6%	A	RF	6, 9	Problem Solving	Acceptable
NR1	56.2%	135	RF	7	Problem Solving	Acceptable
MC8	75.0%	B	RF	7, 8	Problem Solving	Acceptable
MC9	56.9%	C	RF	8	Procedural	Acceptable
NR2	54.1%	6.25	RF	9	Problem Solving	Excellence
MC10	75.8%	A	RF	10	Problem Solving	Acceptable
NR3	45.4%	5.95	RF	10	Problem Solving	Acceptable
MC11	86.5%	B	RF	11	Procedural	Acceptable
MC12	68.5%	A	RF	12	Problem Solving	Acceptable
NR4	56.9%	28	RF	12	Procedural	Acceptable

Question	Diff.*	Key	Topic	Outcome	Cognitive Level	Standard
MC13	63.9%	A	RF	13	Conceptual	Acceptable
NR5	81.5%	2.25	RF	14	Conceptual	Acceptable
MC14	52.5%	D	RF	1, 14	Problem Solving	Excellence
MC15	81.8%	C	TRIG	1	Procedural	Acceptable
MC16	71.4%	B	TRIG	2	Conceptual	Acceptable
NR6	39.9%	15	TRIG	2	Problem Solving	Acceptable
MC17	76.7%	D	TRIG	3	Conceptual	Acceptable
MC18	67.2%	A	TRIG	3, 6	Problem Solving	Excellence
MC19	77.0%	B	TRIG	4	Conceptual	Acceptable
MC20	61.9%	A	TRIG	4	Problem Solving	Acceptable
NR7	76.9%	132	TRIG	6	Problem Solving	Acceptable
MC21	47.5%	C	TRIG	6	Procedural	Excellence
MC22	68.6%	B	PCBT	2	Conceptual	Acceptable
MC23	70.5%	D	PCBT	3	Problem Solving	Acceptable
MC24	71.3%	D	PCBT	4	Procedural	Acceptable
NR8	54.1%	34, 43	PCBT	4	Conceptual	Excellence

*Difficulty—percentage of students answering the question correctly

Question	Average Raw Score	Key	Topic	Outcome	Conceptual Level	Standard
WR1	3.2/5	See Sample Solution	RF	1	Conceptual, Procedural	Acceptable, Acceptable
WR2	2.6/5	See Sample Solution	TRIG	5, 1	Procedural, Conceptual	Excellence, Acceptable
WR3	3.1/5	See Sample Solution	PCBT	3, 1, 2	Procedural, Problem Solving	Acceptable, Acceptable

Mathematics 30–1 Diploma Examination January 2019 Form 1 – Released Items

1. The point $P(3, 8)$ is on the graph of $y = b^x$, where $b > 1$. The corresponding point, P', on the graph of $y + 3 = b^{x+1}$ is

A. (2, 11)
B. (2, 5)
C. (4, 11)
D. (4, 5)

Use the following information to answer question 2.

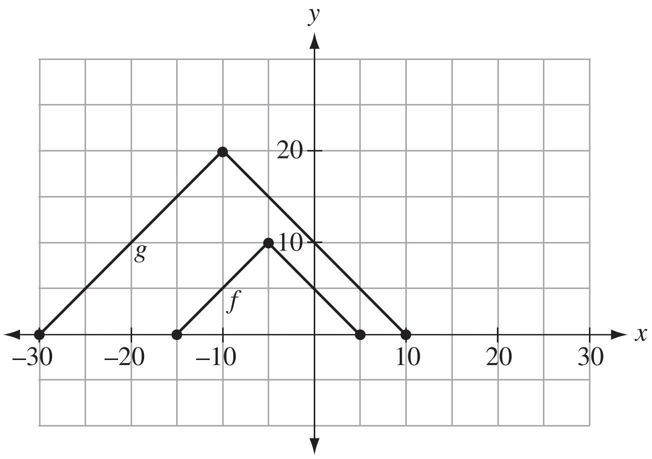
The function $y = f(x)$ has a domain of $\{x \mid 2 \le x \le 6,\ x \in R\}$ and a range of $\{y \mid -4 \le y \le 8,\ y \in R\}$. The function undergoes the transformation $y = -f\left(\frac{1}{2}x\right)$.

2. The domain and range of the transformed function are shown in row

Row	Domain	Range
A.	$\{x \mid -12 \le x \le -4,\ x \in R\}$	$\{y \mid -4 \le y \le 8,\ y \in R\}$
B.	$\{x \mid -6 \le x \le -2,\ x \in R\}$	$\{y \mid -8 \le y \le 16,\ y \in R\}$
C.	$\{x \mid 1 \le x \le 3,\ x \in R\}$	$\{y \mid -8 \le y \le 4,\ y \in R\}$
D.	$\{x \mid 4 \le x \le 12,\ x \in R\}$	$\{y \mid -8 \le y \le 4,\ y \in R\}$

Use the following information to answer question 3.

The graph of $y = f(x)$ is transformed into the graph of $y = g(x)$, shown below.



3. An equation for $g(x)$ in terms of $f(x)$ is

A. $g(x) = f(2x) + 10$

B. $g(x) = f\left(\frac{1}{2}x\right) + 10$

C. $g(x) = 2f(2x)$

D. $g(x) = 2f\left(\frac{1}{2}x\right)$

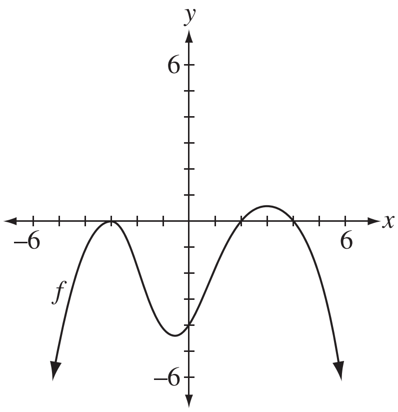
---

4. If Point $A(-3, 4)$ is a point on the graph of $y = f(x)$, then the corresponding image point, A', on the graph of $y = \frac{1}{2}f(3x + 12) - 1$ is

A. (3, 1)
B. (3, 7)
C. (–5, 1)
D. (–5, 7)

Use the following information to answer question 5.

The graph of $y = f(x)$ is shown below.



5. The number of points that would be invariant when the graph of $y = f(x)$ is reflected in the line $y = x$ is

A. 1
B. 2
C. 3
D. 4

---

6. A restriction on the domain of the graph of the quadratic function $f(x) = a(x - c)^2 + d$ that would ensure the inverse of $y = f(x)$ is **always** a function is

A. $x \geq 0$
B. $x \geq a$
C. $x \geq c$
D. $x \geq d$

7. Given the function $f(x) = 4\left(\frac{1}{3}\right)^x - 16$, the y-intercept of the graph of $y = f^{-1}(x)$, to the nearest hundredth, is

A. –1.26
B. –2.52
C. –9.64
D. –12.00

Numerical Response

1. If $\log_2 k = \frac{1}{2}$ and $\csc\theta = k$, where $90° \leq \theta \leq 270°$, then the value of θ, to the nearest degree, is ____________°.

(Record your answer in the numerical-response section on the answer sheet.)

8. If $\log_3 5 = 3y$, $\log_3 4 = 2x$, and $\log_3 m^2 = 6$, then $\log_3\left(100m^4\right)$ is equivalent to

A. $3y + 2x + 6$
B. $6y + 2x + 12$
C. $6y + 2x + 24$
D. $9y^2 + 2x + 36$

9. The expression $\log_a b + 4\log_a(ac) - 4$, where $a, b, c > 1$, written as a single logarithm, is

A. $\log_a\left(\frac{ba^4c^4}{4}\right)$

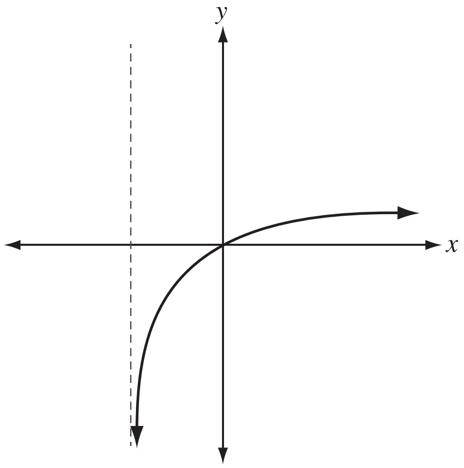
B. $\log_a\left(\frac{bc^4}{a^3}\right)$

C. $\log_a\left(bc^4\right)$

D. $\log_a(bc)$

Use the following information to answer numerical-response question 2.

The graph of $y = \log_b(x + c)$, shown below, passes through the points $(0, 0)$ and $\left(\frac{3}{2}, \frac{1}{2}\right)$.



Numerical Response

2. The value of the base, ***b***, to the nearest hundredth, is ___________.

(Record your answer in the numerical-response section on the answer sheet.)

10. Given $y = \dfrac{a^x}{a^{(2x-6)}}$ and $\log_a y = x$, where $a > 1$, the value of x is

A. 3

B. -3

C. 0 and $\frac{7}{2}$

D. $-\frac{3}{2}$ and 2

Use the following information to answer numerical-response question 3.

Joe invested \$4 500 at a fixed annual interest rate compounded annually. At the end of 12 years, the investment has doubled in value.

Numerical Response

3. To the nearest hundredth of a percent, Joe’s investment pays interest at ______%/year compounded annually.

(Record your answer in the numerical-response section on the answer sheet.)

Use the following information to answer question 11.

In order to factor the polynomial function $p(x) = 3x^3 - 2x^2 - 19x - 6$, a student determined that the function has a zero of $x = 3$. He then wrote the polynomial as a product of a linear factor and a quadratic factor, as shown below, where a, b, and $c \in I$.

$$p(x) = (x + a)(3x^2 + bx + c)$$

11. Which of the following rows shows the correct values for ***a*** and ***b***?

Row	***a***	***b***
A.	–3	–11
B.	–3	7
C.	3	–11
D.	3	7

Use the following information to answer question 12.

A particular polynomial function has the following characteristics.

- A factor of $(x + 2)$ with multiplicity 3
- $P(0) = -24$
- A minimum value of -66

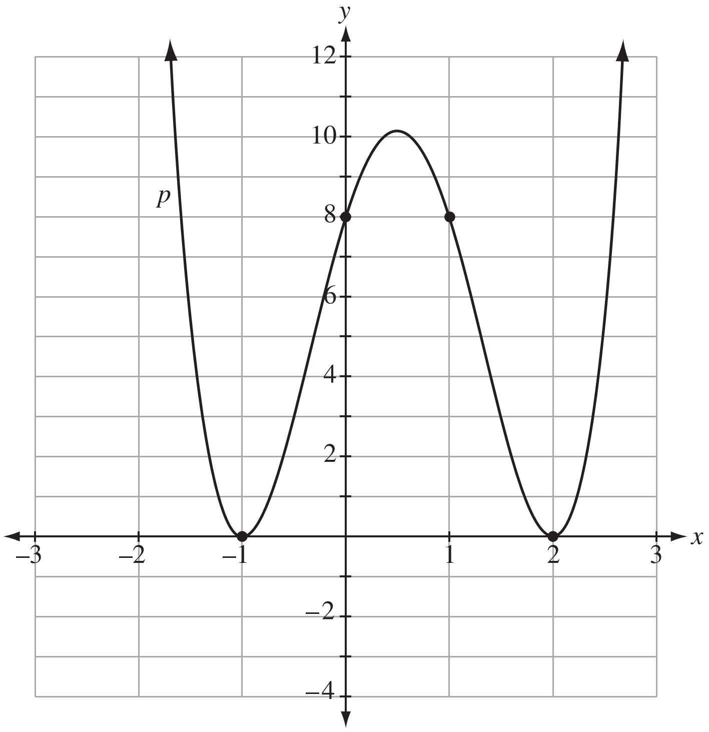
12. *For the polynomial function described above, the minimum possible degree is* ___***i***___ *and the leading coefficient is* ___***ii***___ .

The statement above is completed by the information in row

Row	***i***	***ii***
A.	4	positive
B.	4	negative
C.	5	positive
D.	5	negative

Use the following information to answer numerical-response question 4.

The graph of the polynomial function, shown below, has integral x- and y-intercepts. The equation of the function can be written in the form $p(x) = ax^4 + bx^3 + cx^2 + dx + e$, where $a, b, c, d, e \in I$.



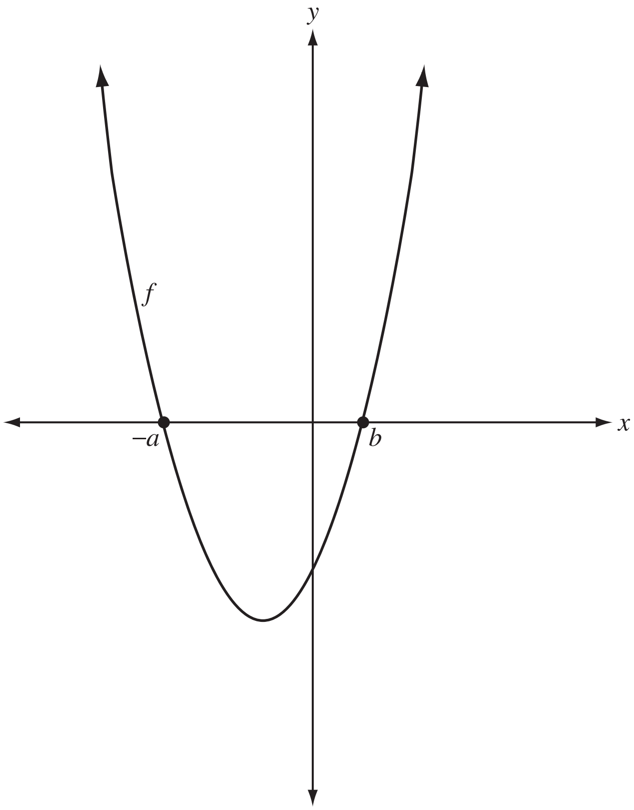
Numerical Response

4. In the equation above, the values of ***a*** and ***e*** are, respectively, _____ and _____.

(Record **both digits** of your answer in the numerical-response section on the answer sheet.)

Use the following information to answer question 13.

The graph of $y = f(x)$ is shown below. The graph of $y = \sqrt{f(x)}$ is to be drawn on the same coordinate plane.



13. *The graph of the function* $y = \sqrt{f(x)}$ *will have a domain of* $\underline{\quad \textbf{\textit{i}} \quad}$, *and there are* $\underline{\quad \textbf{\textit{ii}} \quad}$ *invariant points associated with this transformation.*

The statement above is completed by the information in row

Row	*i*	*ii*
A.	$\{x \mid x \le -a \text{ or } x \ge b, x \in R\}$	4
B.	$\{x \mid x \le -a \text{ or } x \ge b, x \in R\}$	2
C.	$\{x \mid -a \le x \le b, x \in R\}$	4
D.	$\{x \mid -a \le x \le b, x \in R\}$	2

Use the following information to answer numerical-response question 5.

The vertical asymptote of the graph of the rational function $y = \dfrac{x^2 - 3x - 10}{4x^2 - x - 18}$ is defined by the equation $x = k$.

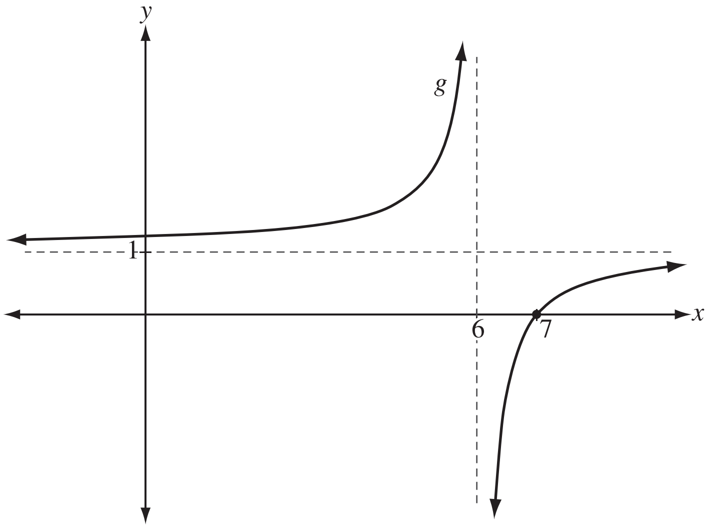
Numerical Response

5. The value of $\boldsymbol{k}$, to the nearest hundredth, is __________.

(Record your answer in the numerical-response section on the answer sheet.)

Use the following information to answer question 14.

The graph of $g(x)$, shown below, can be represented in the form $g(x) = \dfrac{a}{x - b} + c$.



The function $f(x)$ is defined by the equation $f(x) = x^2 + 3x - 4$.

14. The domain of the function $h(x) = (g \circ f)(x)$ will have the restriction

A. $x \neq 6$

B. $x \neq 0$

C. $x \neq -4, 1$

D. $x \neq -5, 2$

Use the following information to answer question 15.

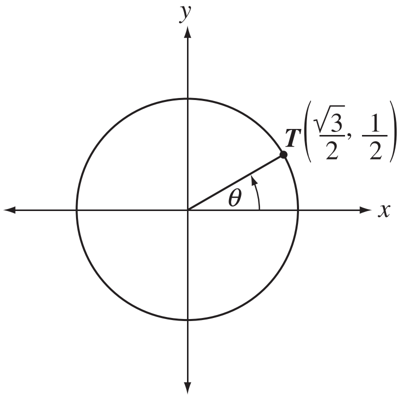
A circular carousel has a diameter of 22 m. An adult is standing on the outer edge of the spinning carousel.

15. The central angle formed when the adult travels 10 m, to the nearest degree, is

A. 26°
B. 38°
C. 52°
D. 63°

Use the following information to answer question 16.

Point $T\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ is the point at the intersection of the unit circle and the terminal arm of angle θ, drawn in standard position, as shown below.



16. If the angle of rotation is changed to become $\theta - \frac{3\pi}{2}$, then the coordinates of the point where the terminal arm intersects the unit circle will be

A. $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

B. $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

C. $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

D. $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Use the following information to answer numerical-response question 6.

The point $(a, -2a)$, where $a > 0$, is the point of intersection between the terminal arm of Angle θ, drawn in standard position, and the unit circle. The value of a can be written in the form $\frac{m}{\sqrt{n}}$, where m and n are single-digit whole numbers.

Numerical Response

6. The values of ***m*** and ***n*** are, respectively, _____ and _____.

(Record **both digits** of your answer in the numerical-response section on the answer sheet.)

17. If $\sec\theta = \frac{13}{12}$, where $\frac{3\pi}{2} \le \theta \le 2\pi$, then the value of $\csc\theta$ is

A. $\frac{5}{13}$

B. $\frac{13}{5}$

C. $-\frac{5}{13}$

D. $-\frac{13}{5}$

Use the following information to answer question 18.

Angles α and β are drawn in standard position with the point $(-7, -4)$ on the terminal arm of Angle α and the point $(2, -1)$ on the terminal arm of Angle β.

18. The **exact** value of $\tan(\alpha + \beta)$ is

A. $\frac{1}{18}$

B. $-\frac{1}{18}$

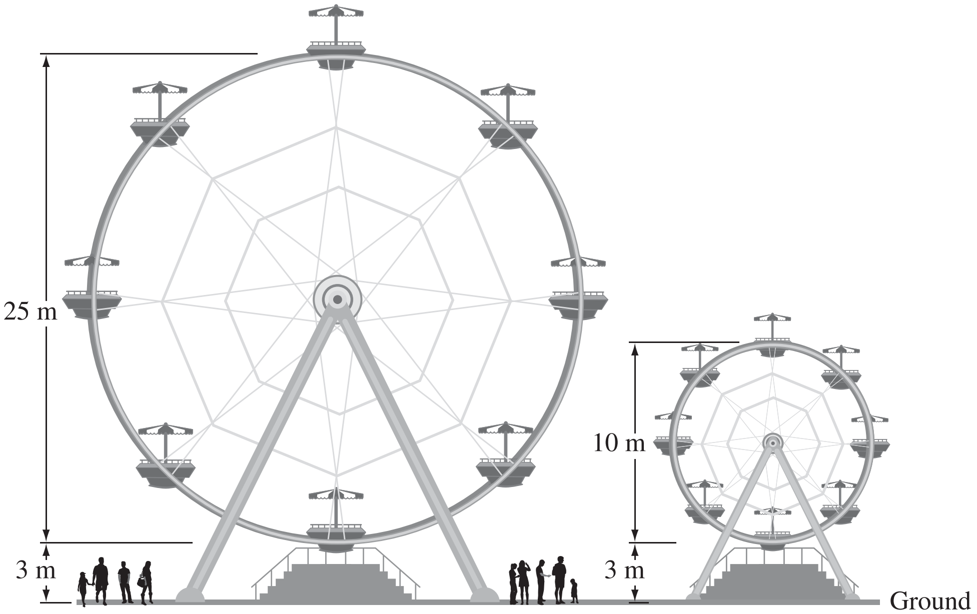
C. $\frac{3}{2}$

D. $-\frac{3}{2}$

Use the following information to answer question 19.

An amusement park has two ferris wheels that both load passengers at the bottom, 3 m above the ground. Both wheels require 40 seconds to complete one revolution. The larger ferris wheel has a diameter of 25 m and the smaller ferris wheel has a diameter of 10 m.

The height of a passenger on each wheel can be expressed in the form $h(t) = a\cos\left[b(t - c)\right] + d$, where $h(t)$ is the height above the ground in metres t seconds after the ride begins.



19. The two parameters that **must** be different in the two functions are

A. a and b

B. a and d

C. b and c

D. c and d

Use the following information to answer question 20.

The average temperature of a particular town in Alberta can be represented by the function $T = 15.8\sin\left(\frac{\pi}{6}(t-3)\right) + 5$, where T is the average temperature in degrees Celsius and t is the time in months after January 1.

20. The minimum number of months, to the nearest tenth of a month, it takes for the average temperature of the town to rise from 5 °C to 15 °C is

A. 1.3 months
B. 2.6 months
C. 3.0 months
D. 4.3 months

Use the following information to answer numerical-response question 7.

Each of the following trigonometric expressions can be simplified to a numerical value, where $\cos\theta \neq 0$ and $\sin\theta \neq 0$.

Expression Number	Trigonometric Expression
1	$\left(\tan^2\theta - \sec^2\theta\right) - 1$
2	$\dfrac{\tan\theta\cos\theta}{\sin\theta}$
3	$\sec\theta - \dfrac{\csc\theta}{\cot\theta}$

Numerical Response

7. Using the expression numbers above, arrange the trigonometric expressions from lowest value to highest value.

Expression Number: ________ , ________ , and ________
Lowest value **Highest value**

(Record all **three digits** of your answer in the numerical-response section on the answer sheet.)

21. The non-permissible values of x for the identity $\csc x \tan x = \sec x$ are

A. $\frac{\pi}{2} + 2n\pi, n \in I$

B. $\frac{\pi}{2} + n\pi, n \in I$

C. $\frac{n\pi}{2}, n \in I$

D. $n\pi, n \in I$

Use the following information to answer question 22.

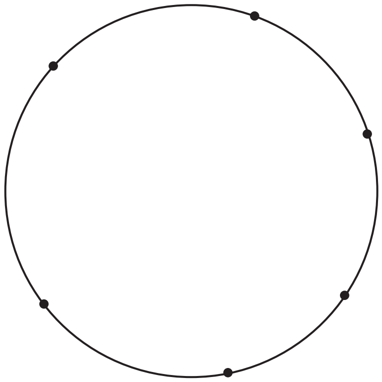
For a team photo, 9 volleyball players are to stand in a line. The photographer will place the shortest player at one of the 2 ends of the line and the tallest player in the middle.

22. The number of possible placements of the players for the photo is

A. 5 040
B. 10 080
C. 20 160
D. 80 640

Use the following information to answer question 23.

A circle is drawn with 6 points on its circumference, as shown in the diagram below.
A student creates polygons with 3 or more sides by connecting the points with straight lines.



23. The number of polygons that can be created that have **at most** 4 sides is

- **A.** 15
- **B.** 20
- **C.** 22
- **D.** 35

24. The coefficient of the ninth term in the expansion of $(3x - 1)^{10}$, written in descending powers of x, is

- **A.** −9
- **B.** −30
- **C.** 135
- **D.** 405

Use the following information to answer numerical-response question 8.

Six statements about the expansion of the binomial $\left(x^2 - \frac{1}{2x}\right)^n$ are shown below.

Statement 1 The expansion contains $n - 1$ terms.
Statement 2 The expansion contains n terms.
Statement 3 The expansion contains $n + 1$ terms.
Statement 4 The largest exponent on the x variable is $2n$.
Statement 5 The largest exponent on the x variable is n.
Statement 6 The largest exponent on the x variable is $n + 2$.

Numerical Response

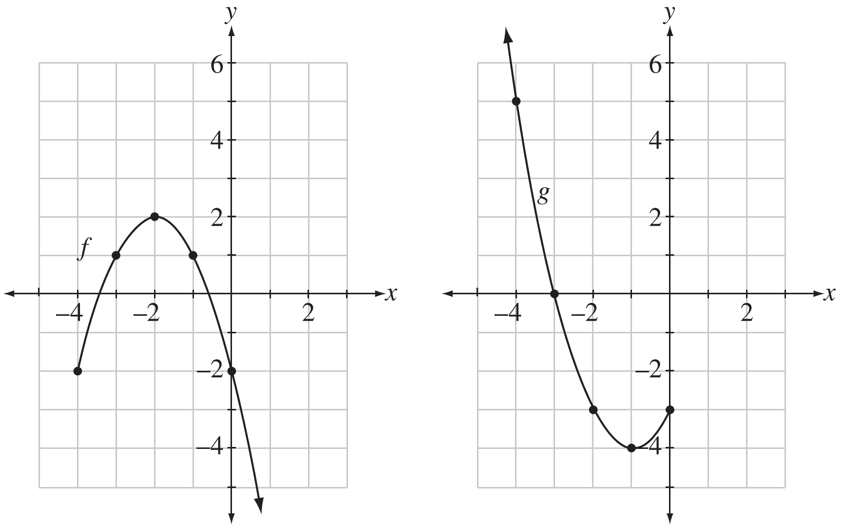
8. The two statements above that are correct are numbered _____ and _____.

(Record **both digits** of your answer **in any order** in the numerical-response section on the answer sheet.)

Written-response question 1 begins on the next page.

Use the following information to answer written-response question 1.

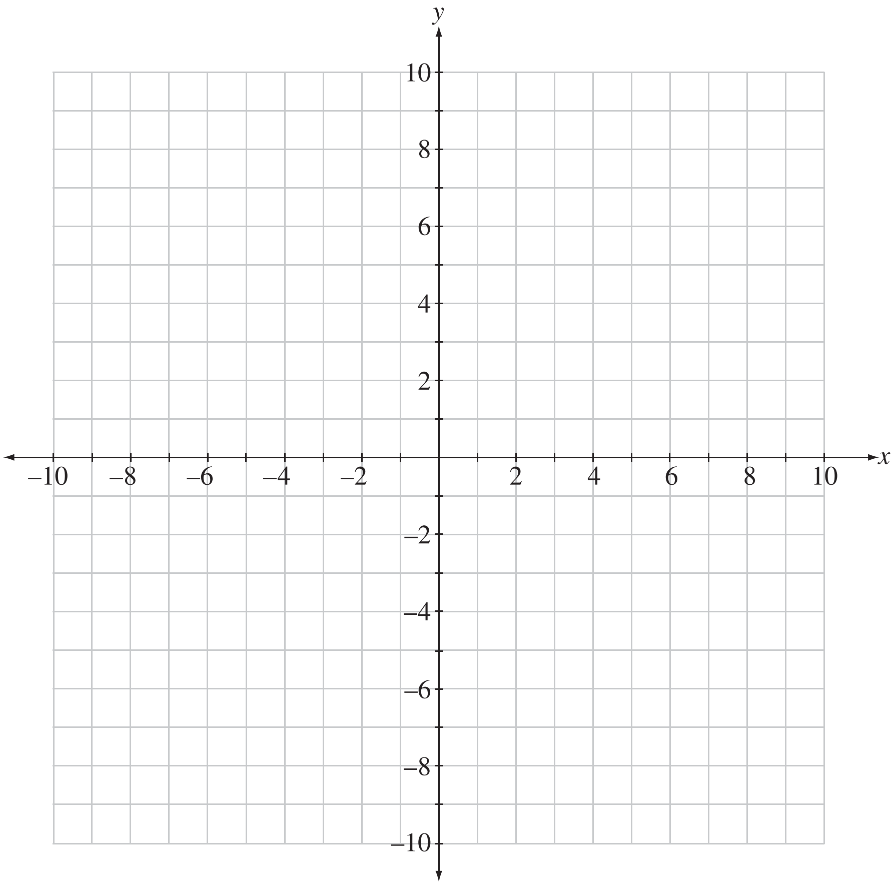
The graphs of two functions, $f(x)$ and $g(x)$, are shown below. Three new functions, $j(x)$, $k(x)$, and $h(x)$, are defined by $j(x) = (f \circ g)(x)$, $k(x) = \dfrac{f(x)}{g(x)}$, and $h(x) = g(x) - f(x)$.



Written Response—5 marks

1. **a.** **Determine** which function, $j(x)$ or $k(x)$, has the larger value when $x = 0$. **[2 marks]**

b. **Sketch** the graph of $h(x)$ on the coordinate plane provided below. Identify the coordinates of the y-intercept, and state the domain of the function. **[3 marks]**



Coordinates of the y-intercept ______________________________

Domain ______________________________

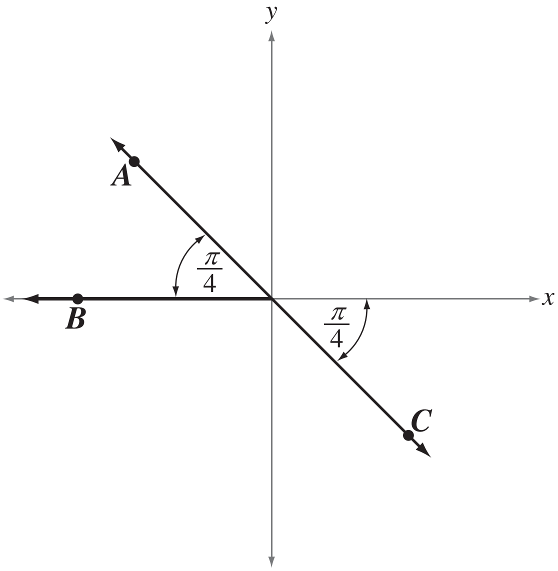
Written-response question 2 begins on the next page.

Written Response—5 marks

2. **a.** **Algebraically solve** the equation $\sec^2\theta + \sec\theta = 2$, where $2\pi \le \theta \le 3\pi$. State the solution as **exact** values. **[3 marks]**

Use the following information to answer the next part of the written-response question.

Points A, B, and C are on the terminal arms of angles drawn in standard position, as shown below. These angles are the solutions for θ to a single trigonometric equation.



b. State the general solution for this equation. **[2 marks]**

Written-response question 3 begins on the next page.

Use the following information to answer written-response question 3.

Students in a math class are creating and exchanging encoded messages with a partner.

Written Response—5 marks

3. **a.** Given that 630 different pairs of students are possible, **algebraically determine** the number of students in the math class. **[3 marks]**

Use the following information to answer the next part of the written-response question.

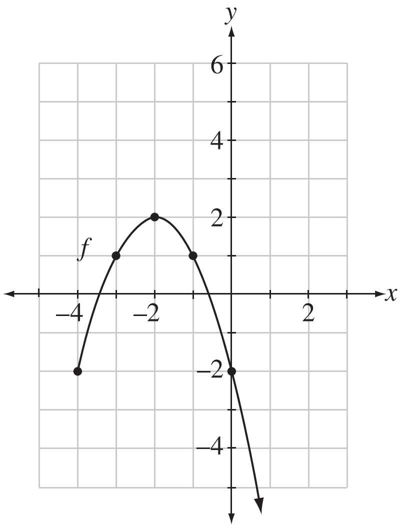
A student is asked to encode the word **FACTOR** by replacing each letter of the word with a different math symbol. There are 10 math symbols available. The student creates a key, a list of his letter-symbol replacements, and gives it to his partner to use to decode the word.

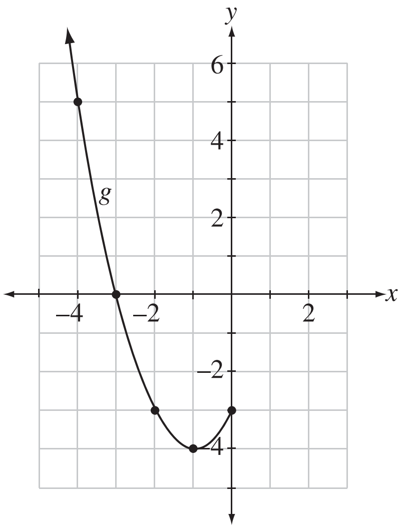
b. **Explain** how you would determine the number of different keys that can be created for the word **FACTOR**, and **determine** the number of possible keys using your strategy. **[2 marks]**

Written-response Question 1
Sample Solution

Use the following information to answer written-response question 1.

The graphs of two functions, $f(x)$ and $g(x)$, are shown below. Three new functions, $j(x)$, $k(x)$, and $h(x)$, are defined by $j(x) = (f \circ g)(x)$, $k(x) = \dfrac{f(x)}{g(x)}$, and $h(x) = g(x) - f(x)$.





Written Response—5 marks

1. **a.** **Determine** which function, $j(x)$ or $k(x)$, has the larger value when $x = 0$. **[2 marks]**

A possible solution to part a

$$j(x) = (f \circ g)(x)$$

$$j(x) = \left(f\left(g(x)\right)\right)$$

$$j(0) = \left(f\left(g(0)\right)\right)$$

$$j(0) = f(-3)$$

$$k(x) = \frac{f(x)}{g(x)}$$

$$k(0) = \frac{f(0)}{g(0)}$$

$$k(0) = \frac{-2}{-3}$$

$$k(0) = \frac{2}{3}$$

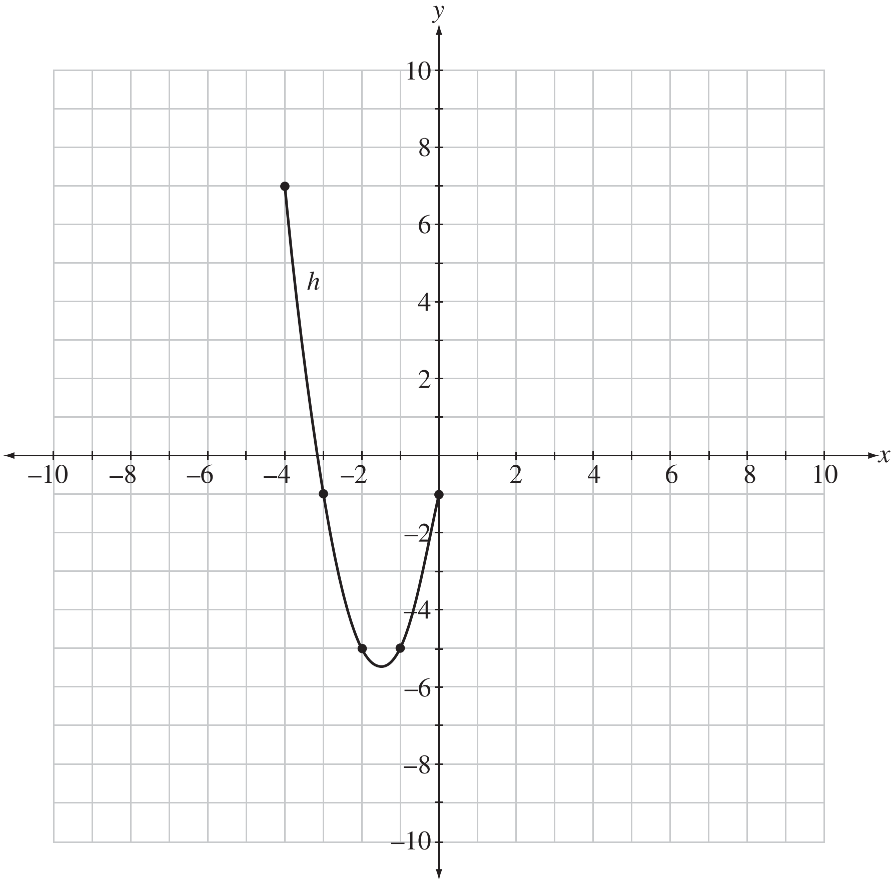
$$j(0) = 1$$

The value of $j(x)$ is larger than the value of $k(x)$ when $x = 0$.

b. **Sketch** the graph of $h(x)$ on the coordinate plane provided below. Identify the coordinates of the y-intercept, and state the domain of the function. **[3 marks]**

A possible solution to part b

x	$g(x)$	$f(x)$	$g(x) - f(x)$	(x, y)
-4	5	-2	7	$(-4, 7)$
-3	0	1	-1	$(-3, -1)$
-2	-3	2	-5	$(-2, -5)$
-1	-4	1	-5	$(-1, -5)$
0	-3	-2	-1	$(0, -1)$



Coordinates of the y-intercept: $(0, -1)$

Domain: $D: \{x \mid -4 \le x \le 0,\ x \in R\}$ or $[-4, 0]$

Note: The domain can be written in either full set notation or interval notation. The y-intercept must be written as an ordered pair to receive full marks.

Specific scoring guide for written-response question 1

Part a

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	The response does not contain valid evidence to support the calculation of the value of either function when $x = 0$. **Note:** A statement that $j(x)$ has the larger value without appropriate supporting evidence will receive a score of 0.
0.5		*For example, the student could* - *provide $f(0)$ or $g(0)$ but not proceed with relevant calculation steps toward $j(0)$ or $k(0)$.*
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student - correctly calculates $j(0)$ or $k(0)$ with supporting evidence OR - provides relevant evidence supporting the calculation of $j(0)$ and $k(0)$ but both calculations are incorrect or incomplete (e.g., the student correctly identifies $f(0)$ and $g(0)$ but correctly calculates $g\left(f(0)\right)$ or $\frac{g(x)}{f(x)}$ instead).
1.5		*For example, the student could* - *provide relevant evidence supporting the calculation of $j(0)$ and $k(0)$ but one calculation is incorrect or incomplete* *OR* - *provide relevant evidence supporting the calculation of both $j(0)$ and $k(0)$ but the same error is correctly carried through both calculations (e.g., an incorrect value of $g(0)$ in both calculations).*
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student - correctly determines, with supporting evidence, which function has the larger value when $x = 0$.

Part b

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	The response does not include a relevant sketch of the graph of $h(x)$ or correct information regarding the *y*-intercept or domain of $h(x)$.
0.5		*For example, the student could* - *identify the correct coodinates of points that exist on the graph of* $h(x)$ *but a graph is not provided* *OR* - *state the correct domain of* $h(x)$.
1	In the response, the student demonstrates minimal mathematical understanding of the problem by applying an appropriate strategy or some relevant mathematical knowledge to complete initial stages of a solution.	In the response, the student - sketches a partially correct graph of $h(x)$ (at least 2 correct features) OR - identifies the correct coordinates of the *y*-intercept and states the correct domain of $h(x)$.
1.5		*For example, the student could* - *sketch a partially correct graph of* $h(x)$ *(2 or 3 correct features)* - *correctly identify the coordinates of the corresponding y-intercept or state the corresponding domain of* $h(x)$.
2	In the response, the student demonstrates good mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student - accurately sketches the graph of $h(x)$ (all 4 features) OR - sketches a partially correct graph of $h(x)$ (2 correct features) - identifies the correct coordinates of the corresponding *y*-intercept and states the correct corresponding domain.
2.5		*For example, the student could* - *accurately sketch the graph of* $h(x)$ *and identify the correct y-intercept or state the correct domain of the function* *OR* - *sketch a partially correct graph of* $h(x)$ *(3 correct features) and identify the correct corresponding y-intercept and domain.*
3	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student - accurately sketches the graph of $h(x)$, identifies the location of the *y*-intercept, and uses appropriate notation to state the correct domain of the function.

Note: Correct features of the sketch are: a parabolic shape, a minimum at approximately $(-1.5, -5.5)$, correct (and clearly indicated) end points, and at least 1 other correctly placed point.

Written-response Question 2
Sample Solution

Written Response—5 marks

2. **a.** **Algebraically solve** the equation $\sec^2\theta + \sec\theta = 2$, where $2\pi \le \theta \le 3\pi$. State the solution as **exact** values. **[3 marks]**

A possible solution to part a

$$\sec^2\theta + \sec\theta - 2 = 0$$

$$(\sec\theta + 2)(\sec\theta - 1) = 0$$

$$\downarrow \qquad \downarrow$$

Therefore, $\sec\theta + 2 = 0$ or $\sec\theta - 1 = 0$

$\sec\theta = -2$ or $\sec\theta = 1$

$\cos\theta = -\frac{1}{2}$ or $\cos\theta = 1$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$ or $\theta = 0$

In the domain $2\pi \le \theta \le 3\pi$,

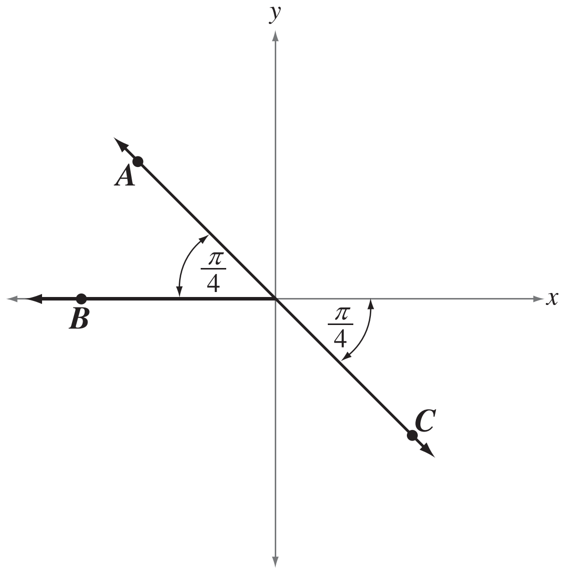
$\theta = \frac{2\pi}{3} + 2\pi$ and $\theta = 0 + 2\pi$

$\theta = \frac{8\pi}{3}$ and $\theta = 2\pi$

The solutions to the equation are $\theta = 2\pi$ and $\theta = \frac{8\pi}{3}$.

Use the following information to answer the next part of the written-response question.

Points A, B, and C are on the terminal arms of angles drawn in standard position, as shown below. These angles are the solutions for θ to a single trigonometric equation.



b. State the general solution for this equation. **[2 marks]**

A possible solution to part b

The three points are on the terminal arms of $\theta = \frac{3\pi}{4}, \pi$ and $\frac{7\pi}{4}$. These angles represent the solutions to the equation in the domain $0 \le \theta \le 2\pi$.

As $\frac{3\pi}{4}$ and $\frac{7\pi}{4}$ have a difference of π, the general solution is:

$$\theta = \frac{3\pi}{4} + n\pi, \ \theta = n\pi + 2n\pi, \ n \in I$$

Note: The general solution can be written as three separate statements.

Specific scoring guide for written-response question 2

Part a

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	The response does not contain valid algebraic steps that would lead to the correct solutions of the equation. **Note:** A correct answer achieved through a non-algebraic process will receive a score of 0.
0.5		*For example, the student could* - *complete one correct step in an attempt to determine the first-degree factors of the equation algebraically but not arrive at the correct factors.*
1	In the response, the student demonstrates minimal mathematical understanding of the problem by applying an appropriate strategy or some relevant mathematical knowledge to complete initial stages of a solution.	In the response, the student - uses an algebraic process to determine the correct first-degree factors of the equation.
1.5		*For example, the student could* - *use an algebraic process to determine the correct first-degree equations (in terms of* $\sec\theta$ *or* $\cos\theta$*) resulting from first-degree factors of the equation.*
2	In the response, the student demonstrates good mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student - uses an algebraic process to determine the correct solutions, in radians, to the equation for the domain $0 \le \theta \le 2\pi$ OR - uses a correct algebraic process to determine the corresponding solutions from incorrect first-degree factors or equations for the domain $2\pi \le \theta \le 3\pi$ OR - uses a correct algebraic process to determine the correct solution from one correct first-degree factor or equation for the domain $2\pi \le \theta \le 3\pi$.
2.5		*For example, the student could* - *use an algebraic process to determine the correct solutions resulting from the correct first-degree equations but include one solution that is incorrect or outside the domain* *OR* - *use a correct algebraic process to determine the correct solutions, in degrees, to the equation for the domain* $360° \le \theta \le 540°$.
3	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student - uses an algebraic process to determine the correct solutions, in radians, to the equation for the domain $2\pi \le \theta \le 3\pi$.

Part b

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	The response does not contain valid components of a general solution for the equation.
0.5		*For example, the student could* • *identify any solution illustrated in the diagram for the domain* $0 \le \theta \le 2\pi$.
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student • creates a general solution for the equation but the statements include an incorrect period, include values that are not solutions, or do not include "$\theta =$" OR • creates a correct general solution for one of the three angles illustrated in the diagram.
1.5		*For example, the student could* • *create a general solution for the equation but a key element of the notation is incorrect (e.g., the number system has been omitted)* *OR* • *create a correct general solution for two of the three angles illustrated in the diagram.*
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student • identifies a correct and complete general solution for the equation.

Written-response Question 3 Sample Solution

Use the following information to answer written-response question 3.

Students in a math class are creating and exchanging encoded messages with a partner.

Written Response—5 marks

3. **a.** Given that 630 different pairs of students are possible, **algebraically determine** the number of students in the math class. **[3 marks]**

A possible solution to part a

$${}_nC_2 = 630$$

$$\frac{n!}{(n-2)!2!} = 630$$

$$\frac{n(n-1)(n-2)!}{(n-2)!2!} = 630$$

$$\frac{n(n-1)}{2} = 630$$

$$n(n-1) = 1\ 260$$

$$n^2 - n - 1\ 260 = 0$$

$$(n-36)(n+35) = 0$$

$$n = 36 \text{ and } n = -35$$

It is not possible to have a negative number of students so $n = -35$ is an extraneous solution. There are 36 students in the math class.

Use the following information to answer the next part of the written-response question.

A student is asked to encode the word **FACTOR** by replacing each letter of the word with a different math symbol. There are 10 math symbols available. The student creates a key, a list of his letter-symbol replacements, and gives it to his partner to use to decode the word.

b. **Explain** how you would determine the number of different keys that can be created for the word **FACTOR**, and **determine** the number of possible keys using your strategy. **[2 marks]**

A possible solution to part b

Each letter is different and needs to be assigned one symbol. Each symbol must be used only once; otherwise, the key will not work.

Possible Solution 1:
The fundamental counting principle can be used to assign a symbol to each letter in a specific order. There are 10 symbols available for the F, 9 symbols available for the A, and so on…

$$\underset{\text{F}}{\underline{10}} \bullet \underset{\text{A}}{\underline{9}} \bullet \underset{\text{C}}{\underline{8}} \bullet \underset{\text{T}}{\underline{7}} \bullet \underset{\text{O}}{\underline{6}} \bullet \underset{\text{R}}{\underline{5}} = 151\ 200$$

There are 151 200 different keys.

Possible Solution 2:
Because the symbols will be arranged in a definite order as they are assigned to the letters, the problem can be solved using permutations. There are 10 symbols in total and 6 of the symbols will be assigned to a letter.

$${}_{10}P_6 = 151\ 200$$

There are 151 200 different keys.

Possible Solution 3:
Because 6 symbols are being chosen from the set of 10, the first part of the problem can be solved using combinations. Once the symbols are chosen, they must then be assigned to a specific letter in the word and the number of possible assignments of the selected symbols can be found using the fundamental counting principle.

$${}_{10}C_6 \times 6! = 151\ 200$$

There are 151 200 different keys.

Specific scoring guide for written-response question 3

Part a

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	The response does not contain valid algebraic solution steps that would lead to the correct number of students in the math class. **Note:** A response with the correct number of students without supporting evidence will receive a score of 0.
0.5		*For example, the student could* • *complete at least one correct step in an algebraic process but not correctly simplify the factorial notation.*
1	In the response, the student demonstrates minimal mathematical understanding of the problem by applying an appropriate strategy or some relevant mathematical knowledge to complete initial stages of a solution.	In the response, the student • completes some correct simplification steps in an algebraic process but does not arrive at the correct quadratic equation; the solution is incomplete.
1.5		*For example, the student could* • *complete correct simplification steps in an algebraic process to determine the correct quadratic equation* *OR* • *complete correct simplification steps in an algebraic process to determine the correct quadratic expression and provide the correct solution using a non-algebraic process.*
2	In the response, the student demonstrates good mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student • completes correct simplification steps in an algebraic process to determine the correct quadratic equation; the solution is incomplete or incorrect due to an error solving the quadratic OR • completes correct simplification steps in an algebraic process to determine the correct quadratic equation; the quadratic equation is correctly solved using a non-algebraic process.
2.5		*For example, the student could* • *determine the correct number of students in the math class using a correct algebraic process but the extraneous solution is not clearly identified or explained.*
3	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student • determines the correct number of students in the math class using a correct algebraic process and clearly identifies and explains the extraneous solution.

Part b

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	The response does not contain a description of a valid solution strategy or the correct number of keys.
0.5		*For example, the student could* • *provide a partial description of a solution strategy that would lead to the correct solution* *OR* • *state the correct number of encryption keys with no supporting evidence.*
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student • provides a full description of a solution strategy that would lead to the correct solution OR • determine the correct number of keys (supported with mathematical evidence).
1.5		*For example, the student could* • *fully describe a solution strategy that would lead to the correct solution and state the number of keys but the answer is not supported with evidence* *OR* • *fully describe a solution strategy that would lead to the correct solution and calculate the number of keys but the answer is incorrect (supporting evidence is correct)* *OR* • *provide a partial description of a solution strategy that would lead to the correct solution* ***and*** *provide the correct number of keys with supporting evidence.*
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student • fully describes a solution strategy that would lead to the correct solution and correctly calculates the number of keys (supported with evidence).

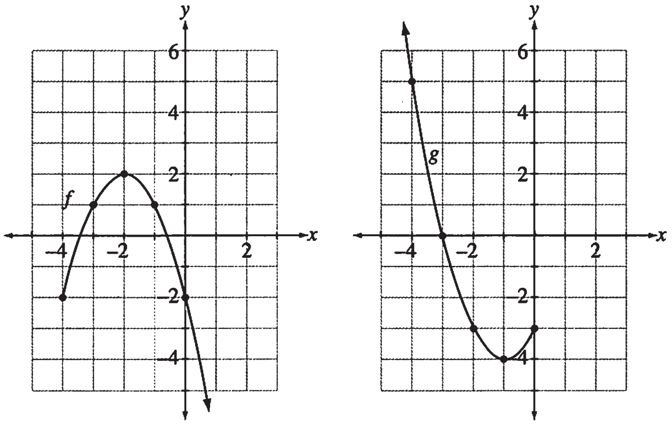
Examples of the Standards for Students’ Work

This section provides sample student responses and scoring rationales as they relate to the general scoring guide. These examples are intended to inform teachers and students of how the scoring guide is applied to specific questions and to provide examples of Mathematics 30-1 work that meet or exceed the acceptable standard for student achievement. Teachers and students should note that directing words are bolded in written-response questions on diploma examinations. A list of these directing words and definitions can be found in the *Mathematics 30-1 Information Bulletin*.

Sample response 1

Use the following information to answer written-response question 1.

The graphs of two functions, $f(x)$ and $g(x)$, are shown below. Three new functions, $j(x)$, $k(x)$, and $h(x)$, are defined by $j(x) = (f \circ g)(x)$, $k(x) = \dfrac{f(x)}{g(x)}$, and $h(x) = g(x) - f(x)$.



Written Response—5 marks

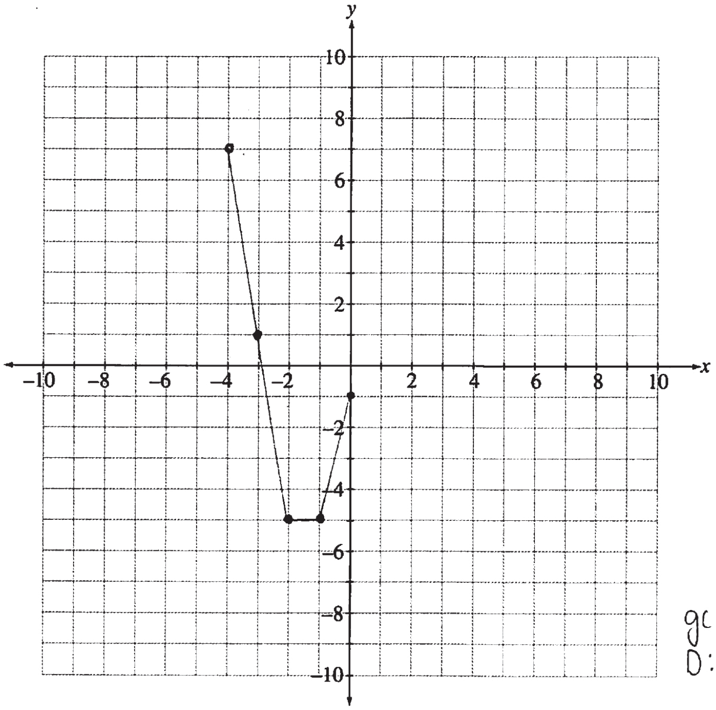
1. **a.** **Determine** which function, $j(x)$ or $k(x)$, has the larger value when $x = 0$. **[2 marks]**

$$\begin{aligned} j(x) &= (f \circ g)(x) \\ &= f(g(x)) \\ &= f(g(0)) \\ &= f(-3) \\ &= 1 \end{aligned}$$

$$\begin{aligned} k(x) &= \frac{f(x)}{g(x)} \\ &= \frac{f(0)}{g(0)} \\ &= \frac{-2}{-3} \\ &= \frac{2}{3} \end{aligned}$$

when x is substituted with 0, $j(x)$ which is a composition of $g(x)$ into $f(x)$ has the larger value of 1 compared to the function of $k(x)$ which is a division of $f(x)$ by $g(x)$ and gives the value of $2/3$. $\therefore$ $j(x)$ has the larger value when $x=0$.

b. **Sketch** the graph of $h(x)$ on the coordinate plane provided below. Identify the coordinates of the y-intercept, and state the domain of the function. **[3 marks]**



$h(x) = g(x) - f(x)$

$g(x)$:
D: $(-\infty, 0]$

$f(x)$:
D: $[-4, \infty)$

$h(x)$:
D: $[-4, 0]$

Coordinates of the y-intercept $(0, -1)$

Domain $\{x \mid -4 \leq x \leq 0, x \in \mathbb{R}\}$

x	$g(x)$	$f(x)$	$h(x) = g(x) - f(x)$
-4	5	-2	$5 - (-2) = 7$
-3	0	1	$0 - (-1) = 1$
-2	-3	2	$-3 - 2 = -5$
-1	-4	1	$-4 - 1 = -5$
0	-3	-2	$-3 - (-2) = -1$
1	undefined		
2	undefined		

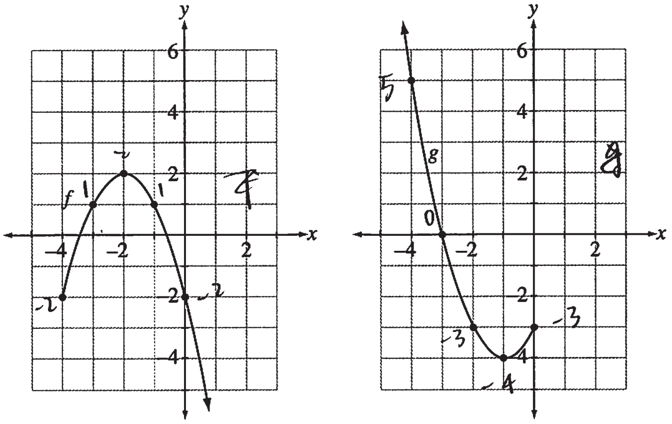
y-int: $(0, -1)$

Total Score - 4 marks	**Rationale**
Part a: 2 marks **Part b: 2 marks**	In part a, the complete and correct calculations for $j(0)$ and $k(0)$ are provided. The response also clearly indicates which function is larger when $x = 0$. In part b, a sketch of $h(x)$ with 2 correct features (correct and clearly indicated endpoints and at least 1 other correctly placed point) is provided. The response also contains the correct coordinates of the y-intercept and domain.

Sample response 2

Use the following information to answer written-response question 1.

The graphs of two functions, $f(x)$ and $g(x)$, are shown below. Three new functions, $j(x)$, $k(x)$, and $h(x)$, are defined by $j(x) = (f \circ g)(x)$, $k(x) = \frac{f(x)}{g(x)}$, and $h(x) = g(x) - f(x)$.



Written Response—5 marks

1. **a.** **Determine** which function, $j(x)$ or $k(x)$, has the larger value when $x = 0$. **[2 marks]**

$j(x) = (f \circ g)x$

$= f(g(x))$

$j(0) = f(g(0))$

$= f(-3)$

$= 1$

$k(x) = \frac{f(x)}{g(x)}$

$k(0) = \frac{f(0)}{g(0)}$

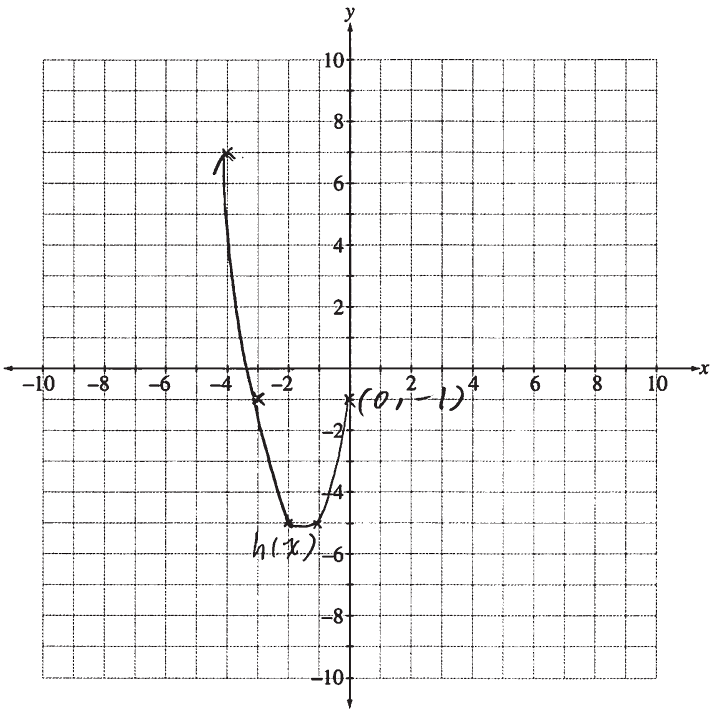
$= \frac{-2}{-3}$

$= \frac{2}{3}$

a. $\because 1 > \frac{2}{3}$

$\therefore j(0) > k(0)$

b. **Sketch** the graph of $h(x)$ on the coordinate plane provided below. Identify the coordinates of the y-intercept, and state the domain of the function. **[3 marks]**



$-3-(-2)=-1$

$-4-1=-5$

$-3-2=-5$

$0-1=-1$

$5-(-2)=7$

Coordinates of the y-intercept $(0, -1)$

Domain $\{x \mid x \le 0,\ x \in \mathbb{R}\}$

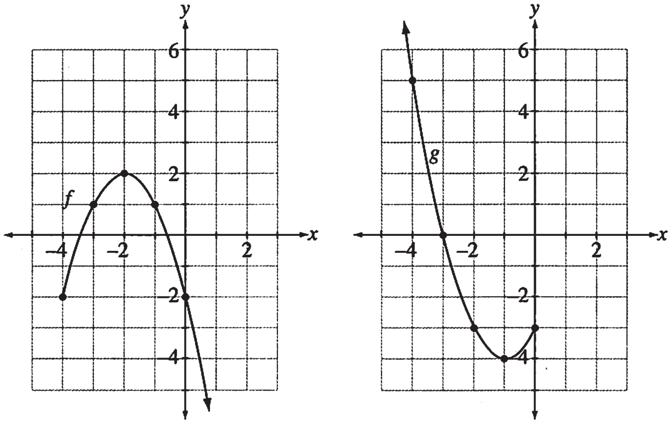
$h(x) = g(x) - f(x)$

Total Score - 4.5 marks	Rationale
Part a: 2 marks **Part b: 2.5 marks**	In part a, the complete and correct calculations for $j(0)$ and $k(0)$ are provided. The response also clearly indicates which function is larger when $x = 0$. In part b, a sketch of $h(x)$ with 3 correct features is provided. Even though the response contains the correct corresponding coordinates of the y-intercept and domain, the sketch does not illustrate two correct endpoints, so the response does not receive full marks.

Sample response 3

Use the following information to answer written-response question 1.

The graphs of two functions, $f(x)$ and $g(x)$, are shown below. Three new functions, $j(x)$, $k(x)$, and $h(x)$, are defined by $j(x) = (f \circ g)(x)$, $k(x) = \frac{f(x)}{g(x)}$, and $h(x) = g(x) - f(x)$.



Written Response—5 marks

1. **a.** **Determine** which function, $j(x)$ or $k(x)$, has the larger value when $x = 0$. **[2 marks]**

$j(x) = f(g(x))$

$j(0) = -3$

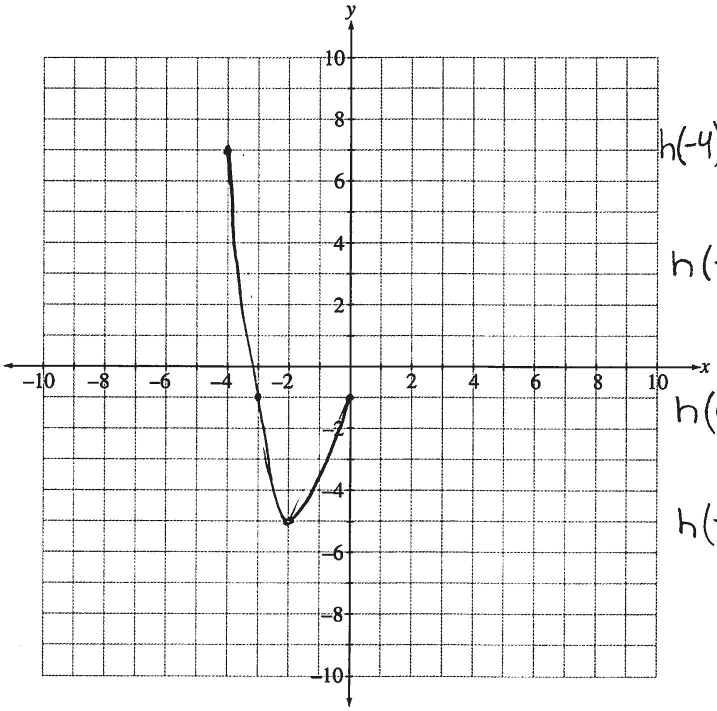
$k(x) = \frac{f(x)}{g(x)}$

$k(0) = \frac{-2}{-3}$

$\boxed{= \frac{2}{3}}$

k(x) has a larger value at x = 0 because it's positive.

b. **Sketch** the graph of $h(x)$ on the coordinate plane provided below. Identify the coordinates of the y-intercept, and state the domain of the function. **[3 marks]**



$g(x) - f(x)$

$h(-4) = 5 - (-2)$
$= 7$

$h(-2) = -3 - 2$
$= -5$

$h(0) = -3 - (-2)$
$= -1$

$h(-3) = 0 - 1$
$= -1$

Coordinates of the y-intercept $(0, -1)$

Domain $\{x \mid -4 \le x \le 0\}$

Total Score - 2.5 marks	Rationale
Part a: 1 mark **Part b: 1.5 marks**	In part a, the response contains the correct calculation for $k(0)$ with supporting evidence. In part b, a sketch of $h(x)$ with 3 correct features is provided. The response also contains the correct coordinates of the y-intercept but, as the domain is incomplete, the response does not meet the criteria for the benchmark score of 2.

Sample response 4

Written Response—5 marks

2. **a.** **Algebraically solve** the equation $\sec^2\theta + \sec\theta = 2$, where $2\pi \le \theta \le 3\pi$. State the solution as **exact** values. **[3 marks]**

$\sec^2\theta + \sec\theta = 2$

$\sec^2\theta + \sec\theta - 2 = 0$

$(\sec\theta + 2)\ (\sec\theta - 1) = 0$

$\sec\theta = -2$

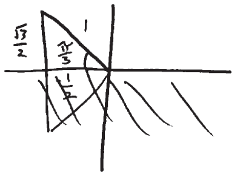
$\frac{1}{\cos\theta} = -2$

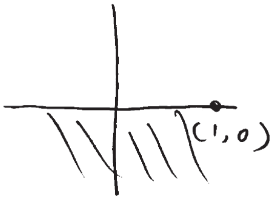
$\cos\theta = \frac{1}{-2}$

$\sec\theta = 1$

$\frac{1}{\cos\theta} = 1$

$1 = \cos\theta$





$\theta = \frac{2\pi}{3} + 2\pi$

$\theta = \frac{8\pi}{3}$

$\theta = 2\pi$

Verify

$\sec^2\theta + \sec\theta = 2$

$\sec^2\theta + \sec\theta$ LS	$= 2$ RS
$\frac{1}{(\cos\frac{8\pi}{3})^2} + \frac{1}{\cos\frac{8\pi}{3}}$	2
$= 1 + 1$	$= 2$
	$= 2$
$= 2$	

LS = RS

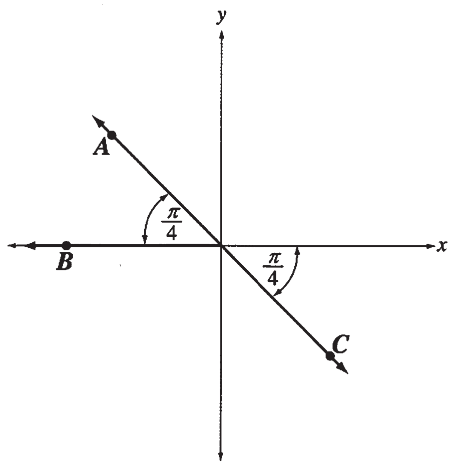
LS	RS
$= \frac{1}{(\cos 2\pi)^2} + \frac{1}{\cos 2\pi}$	$= 2$
$= 1 + 1$	$= 2$
$= 2$	$= 2$

LS = RS

$\theta = 2\pi, \frac{8\pi}{3}$

Use the following information to answer the next part of the written-response question.

Points A, B, and C are on the terminal arms of angles drawn in standard position, as shown below. These angles are the solutions for θ to a single trigonometric equation.



b. State the general solution for this equation. **[2 marks]**

Point A . $\theta = \pi - \theta_r$ PoinB : $\theta = \pi$

$\theta = \pi - \frac{\pi}{4}$

$\theta = \frac{3\pi}{4}$

Point C : $\theta = 2\pi - \frac{\pi}{4}$

$\theta = \frac{7\pi}{4}$

Point A : $\theta = \frac{3\pi}{4} + 2n\pi , n \in I$

Point B : $\theta = \pi + 2n\pi , n \in I$

Point C : $\theta = \frac{7\pi}{4} + 2n\pi , n \in I$

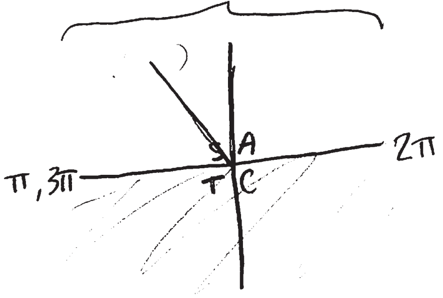
Total Score - 5 marks	Rationale
Part a: 3 marks **Part b: 2 marks**	In part a, the response contains the correct solutions to the first-degree factors of the given trigonometric equation for the domain $2\pi \le \theta \le 3\pi$. In part b, the response addresses all 3 angles in the diagram and includes a correct general solution for each using the proper notation.

Sample response 5

Written Response—5 marks

2. **a.** **Algebraically solve** the equation $\sec^2\theta + \sec\theta = 2$, where $2\pi \le \theta \le 3\pi$. State the solution as **exact** values. **[3 marks]**

α = reference angle



$2 = \sec^2\theta + \sec\theta$

$0 = \sec^2\theta + \sec\theta - 2$

$0 = (\sec\theta + 2)(\sec\theta - 1)$

$\sec\theta = -2$, $\sec\theta = 1$

$\frac{1}{\cos\theta} = -2$, $\frac{1}{\cos\theta} = 1$

$\cos\theta = -\frac{1}{2}$, $\cos\theta = 1$

$\alpha = \cos(-\frac{1}{2})$, $\alpha = \cos(1)$

$\alpha = \frac{\pi}{3}$, $\alpha = 0$

$\theta = 0 + 2\pi$, $0 + 2\pi + \pi$

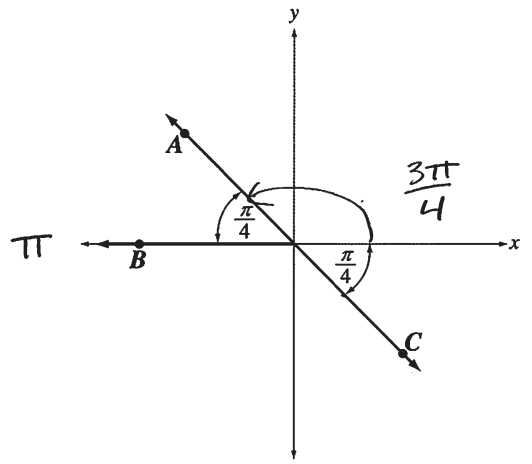
$\boxed{\theta = 2\pi, 3\pi}$

$\theta = \frac{2\pi}{3} + 2\pi$

$\boxed{\theta = \frac{8\pi}{3}}$

Use the following information to answer the next part of the written-response question.

Points A, B, and C are on the terminal arms of angles drawn in standard position, as shown below. These angles are the solutions for θ to a single trigonometric equation.



b. State the general solution for this equation. **[2 marks]**

$$\theta = \frac{3\pi}{4} \pm n\pi, n \in I$$

$$\theta = \pi \pm 2n\pi, n \in I$$

Total Score - 4.5 marks	Rationale
Part a: 2.5 marks **Part b: 2 marks**	In part a, the correct solutions to the first-degree factors of the given trigonometric equation for the domain $2\pi \leq \theta \leq 3\pi$ are provided. The inclusion of an incorrect value in the final answer indicates that the student does not have complete mathematical understanding of the problem. In part b, the response includes a general solution that addresses all 3 angles in the diagram using the correct notation.

Sample response 6

SOH CAH TOA

Written Response—5 marks

2. **a.** **Algebraically solve** the equation $\sec^2\theta + \sec\theta = 2$, where $\boxed{2\pi \le \theta \le 3\pi.}$ State the solution as **exact** values. **[3 marks]**

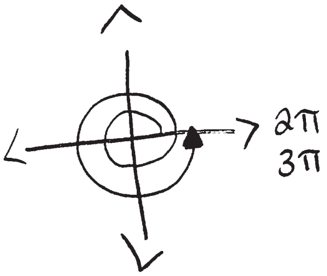
$\sec^2\theta + \sec\theta = 2$

$\sec^2\theta + \sec\theta - 2 = 0$

$(\sec\theta - 1)(\sec\theta + 2)$

$\theta = 1$

$\theta = -2$



$\cos\theta \rightarrow$

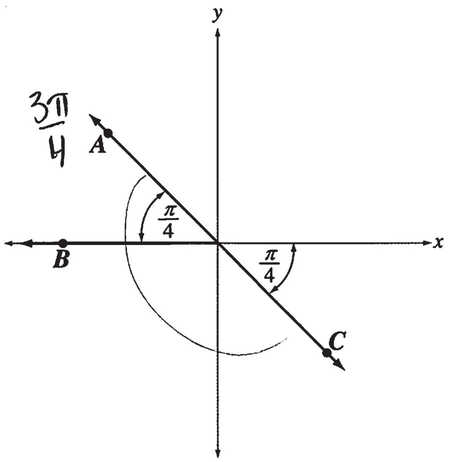
$\theta = 1$

$\theta = -\frac{1}{2}$

$$\boxed{\begin{array}{l} \theta = 2\pi, 3\pi \\ \theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3} \end{array}}$$

Use the following information to answer the next part of the written-response question.

Points A, B, and C are on the terminal arms of angles drawn in standard position, as shown below. These angles are the solutions for θ to a single trigonometric equation.



b. State the general solution for this equation. **[2 marks]**

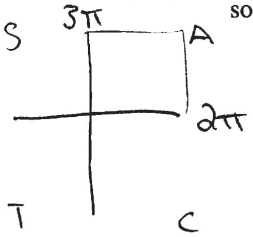
$$\boxed{\theta = \frac{3\pi}{4} + \pi N,\ N \in Z}$$

Total Score - 3.5 marks	Rationale
Part a: 2 marks **Part b: 1.5 marks**	In part a, the correct solutions to the first-degree factors of the given trigonometric equation for the domain $2\pi \leq \theta \leq 3\pi$ are provided. However, because the solution also includes values not in the required domain and an incorrect value within the required domain, the response cannot be awarded more than the benchmark score of 2. In part b, the response includes a general solution that addresses only 2 of the angles in the diagram.

Sample response 7

Written Response—5 marks

2. **a.** **Algebraically solve** the equation $\sec^2\theta + \sec\theta = 2$, where $2\pi \le \theta \le 3\pi$. State the solution as **exact** values. **[3 marks]**



$\sec\theta = \dfrac{1}{\cos\theta}$

let $A = \sec\theta$

$A^2 + A = 2$
$-2 \quad -2$

$A^2 + A - 2 = 0$

$(A+2)(A-1) = 0$

$A = -2 \quad A = 1$

$\sec\theta = -2$

$\downarrow$

$\left(\dfrac{1}{\cos\theta}\right)^{\times\cos\theta} = (-2)^{\times\cos\theta}$

$\dfrac{1}{-2} = \dfrac{-2(\cos\theta)}{-2}$

$\cos\theta = \dfrac{-1}{2}$

$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$ → angle doesn't exist in the restriction so it's not a solution

$\sec\theta = 1$

$\downarrow$

$\left(\dfrac{1}{\cos\theta}\right)^{\times\cos\theta} = (1)^{\times\cos\theta}$

$1 = 1(\cos\theta)$

$\cos\theta = 1$

$\theta = 2\pi n, n \in \mathbb{Z}$

$\boxed{\text{Solution} = 2\pi}$

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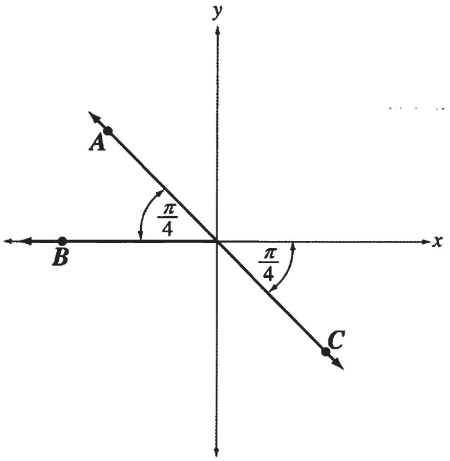
Use the following information to answer the next part of the written-response question.

Points A, B, and C are on the terminal arms of angles drawn in standard position, as shown below. These angles are the solutions for θ to a single trigonometric equation.



b. State the general solution for this equation. **[2 marks]**

$\frac{\pi}{4} = \text{ref} \angle$

$\angle AB = \frac{4\pi}{4} - \frac{\pi}{4} = \frac{3\pi}{4}$

$\angle C: \frac{8\pi}{4} - \frac{\pi}{4} = \frac{7\pi}{4}$

$\frac{7\pi}{4} - \frac{3\pi}{4} = \frac{4\pi}{4}$ or π

$$\boxed{\theta: \frac{3\pi}{4} + \pi n, n \in \mathbb{Z}}$$

Total Score - 3.5 marks	**Rationale**
Part a: 2 marks **Part b: 1.5 marks**	In part a, only the correct solutions to the first-degree factors of the given trigonometric equation for the domain $0 \leq \theta \leq 2\pi$ are provided. In part b, the response includes a general solution that addresses only 2 of the angles in the diagram.

Sample response 8

Use the following information to answer written-response question 3.

Students in a math class are creating and exchanging encoded messages with a partner.

Written Response—5 marks

3. **a.** Given that 630 different pairs of students are possible, **algebraically determine** the number of students in the math class. **[3 marks]**

Let the number of students in the math class be n.

To find the number of possible different pairs of 2, choose 2 from n using combinations,

There are 630 different possible pairs of 2:

$$_nC_2 = 630$$

$$_nC_r = \frac{n!}{(n-r)!r!}$$

$$_nC_2 = \frac{n!}{(n-2)!\,2!} = 630$$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}\,2!} = 630$$

$$\frac{n(n-1)}{2!} = 630$$

$$n(n-1) = 630 \times 2!$$

$$n(n-1) = 1260$$

$$n^2 - n - 1260 = 0$$

$$(n-36)(n+35) = 0$$

$$n = 36 \text{ or } n = -35$$

↑ this does not work for the context because there cannot be a negative number of students.

$\boxed{\therefore \text{ There 36 students in the math class.}}$

Use the following information to answer the next part of the written-response question.

A student is asked to encode the word **FACTOR** by replacing each letter of the word with a different math symbol. There are 10 math symbols available. The student creates a key, a list of his letter-symbol replacements, and gives it to his partner to use to decode the word.

b. **Explain** how you would determine the number of different keys that can be created for the word **FACTOR**, and **determine** the number of possible keys using your strategy. **[2 marks]**

There are 10 math symbols to choose from, and there are 6 letters in the word FACTOR

∴ 6 symbols need to be chosen.

At this point, I am only choosing 6 symbols from 10, and the order I choose them in does not matter.

to find the number of possible ways of choosing 6 items out of 10, do $_{10}C_6$.

After choosing the letters, I need to arrange them to each represent a letter in the word FACTOR.

Because each letter is unique, the order I arrange the symbols now matter.

· I will arrange the six symbols so each one matches a unique letter. To find possible arrangements for 6 items, do $_6P_6$

Because I need to choose the letters and arange them, I multiply the number of possible outcomes for the 2 together: $_{10}C_6 \times {}_6P_6$ and get the final answer.

$$_{10}C_6 \times {}_6P_6 = 151200$$

Therefore, there are 151200 possible different keys that can be created.

Total Score - 5 marks	Rationale
Part a: 3 marks **Part b: 2 marks**	In part a, the response illustrates a valid algebraic process to determine the correct number of students in the class. The response also clearly identifies and explains the extraneous solution. In part b, the response includes a full explanation which addresses the requirement to select and arrange 6 unique math symbols and the correct number of keys.

Sample response 9

Use the following information to answer written-response question 3.

Students in a math class are creating and exchanging encoded messages with a partner.

Written Response—5 marks

3. **a.** Given that 630 different pairs of students are possible, **algebraically determine** the number of students in the math class. **[3 marks]**

$n!$

$$630 = \frac{n!}{(n-2)!\,2!} = \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}\,2!}$$

$$630 = \frac{n(n-1)}{2!}$$

$$2(630) = \left(\frac{n(n-1)}{2}\right)2$$

$$1260 = n(n-1)$$

$$= n^2 - n - 1260$$

$$x = \frac{-1 \pm \sqrt{-1^2 - 4(-1)(-1260)}}{2(-1)}$$

$$\frac{1 \pm \sqrt{5041}}{-2}$$

$$\frac{1 \pm 71}{-2}$$

$$\boxed{n = 36} \quad n = -36$$

cannot have negative students

36 students in the Math class

Use the following information to answer the next part of the written-response question.

A student is asked to encode the word **FACTOR** by replacing each letter of the word with a different math symbol. There are 10 math symbols available. The student creates a key, a list of his letter-symbol replacements, and gives it to his partner to use to decode the word.

b. **Explain** how you would determine the number of different keys that can be created for the word **FACTOR**, and **determine** the number of possible keys using your strategy. **[2 marks]**

- Put 6 lines to represent each letter
- Write 10, 9, 8, 7, 6, 5 in each spot, representing how many math symbols he can choose from for each letter of FACTOR.
- Multiply them all together, that is how many possible keys there can be.

$$\underset{F}{\underline{10}} \times \underset{A}{\underline{9}} \times \underset{C}{\underline{8}} \times \underset{T}{\underline{7}} \times \underset{O}{\underline{6}} \times \underset{R}{\underline{5}} = 151200 \text{ keys}$$

Total Score - 3 marks	Rationale
Part a: 2 marks **Part b: 1 mark**	In part a, the response includes valid algebraic steps to determine the correct resulting quadratic equation but the solution is incorrect due to an error in solving the quadratic. In part b, the response includes the correct number of keys. Even though the explanation describes the mechanics of finding the number of keys, the student does not link order and non-repetition to the numbers used in the calculation.

Sample response 10

Use the following information to answer written-response question 3.

Students in a math class are creating and exchanging encoded messages with a partner.

Written Response—5 marks

3. **a.** Given that 630 different pairs of students are possible, **algebraically determine** the number of students in the math class. **[3 marks]**

$$_nC_r = \frac{n!}{(n-r)!\,r!}$$

$$_nC_2 = \frac{n!}{(n-2)!\,2!}$$

$$2 \cdot 630 = \frac{n!}{(n-2)!\,\cancel{2}\ \cancel{\cdot 2}}$$

$$1260 = \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}}$$

$$1260 = n(n-1)$$

$$\cancel{1260} = n^2 - n - 1260$$
$$-\cancel{1260}$$

$$0 = n^2 - n - 1260$$

P: -1260
S: -1
F: $-36, 35$

$$0 = (n-36)(n+35)$$

$$\boxed{n = 36} \qquad \cancel{n = -35} \Leftarrow \text{Can't have a negative amount of students}$$

24

There are 36 students in the math class.

Use the following information to answer the next part of the written-response question.

A student is asked to encode the word **FACTOR** by replacing each letter of the word with a different math symbol. There are 10 math symbols available. The student creates a key, a list of his letter-symbol replacements, and gives it to his partner to use to decode the word.

b. **Explain** how you would determine the number of different keys that can be created for the word **FACTOR**, and **determine** the number of possible keys using your strategy. **[2 marks]**

There are 10 possible math symbols that can be used to make the code. The word FACTOR has 6 different letters. This means that each seperate key would only use 6 out of the 10 math symbols.

By using ${}_{10}C_6$ you can determine the number of different keys that can be created.

${}_{10}C_6 = 210.$ There are 210 different key possibilities.

Total Score - 3.5 marks	Rationale
Part a: 3 marks **Part b: 0.5 marks**	In part a, the response illustrates a valid algebraic process to determine the correct number of students in the class. The response also clearly identifies and explains the extraneous solution. In part b, the response contains an explanation that only addresses the selection of 6 symbols and an incorrect calculation for the number of keys.

Sample response 11

Use the following information to answer written-response question 3.

Students in a math class are creating and exchanging encoded messages with a partner.

Written Response—5 marks

3. **a.** Given that 630 different pairs of students are possible, **algebraically determine** the number of students in the math class. **[3 marks]**

Let x = amount of students

$_xC_2 = 630$

$_nC_r = \frac{n!}{(n-r)!r!}$

$\frac{x!}{(x-2)!2!} = 630$

$\frac{x!}{(x-2)!\cancel{2}} \times \cancel{2} = 630 \times 2$

$\frac{x!}{(x-2)!} = 1260$

$\frac{x(x-1)\cancel{(x-2)!}}{\cancel{(x-2)!}} = 1260$

$x(x-1) = 1260$

$x^2 - x = 1260$

$x^2 - x - 1260 = 0$

$(x-36)(x+35) = 0$

$x - 36 = 0$

$x = 36$ ✓

$x + 35 = 0$

$x = -35$

× because no value in $_nC_r$ can be negative

$\boxed{x = 36}$

Verify:

$_xC_2 = 630$ $x = 36$

$_{36}C_2 = 630$

$630 = 630$

Use the following information to answer the next part of the written-response question.

A student is asked to encode the word **FACTOR** by replacing each letter of the word with a different math symbol. There are 10 math symbols available. The student creates a key, a list of his letter-symbol replacements, and gives it to his partner to use to decode the word.

b. **Explain** how you would determine the number of different keys that can be created for the word **FACTOR**, and **determine** the number of possible keys using your strategy. **[2 marks]**

6 math symbols for 6 letters, 4 will not be used

- So you have $_{10}C_6$ as the different combinations of symbols you can use (210 combinations)
- Of those 6 letters there are $_6P_6$ different ways in which you can have represent a letter (720 ways)

Overal:

$_{10}C_6 \times {}_6P_6 = \text{total}$

$\text{total} = 210 \times 720$

$\boxed{\text{total} = 151200}$

Total Score - 4.5 marks	Rationale
Part a: 3 marks **Part b: 1.5 marks**	In part a, the response illustrates a valid algebraic process to determine the correct number of students in the class. The response also clearly identifies and explains the extraneous solution. In part b, the response includes a partial explanation and the correct number of keys. The explanation does not explicitly address the fact that symbols cannot be repeated.

Sample response 12

Use the following information to answer written-response question 3.

Students in a math class are creating and exchanging encoded messages with a partner.

2 people

Written Response—5 marks

3. **a.** Given that 630 different pairs of students are possible, **algebraically determine** the number of students in the math class. **[3 marks]**

$$ {}_nC_r = \frac{n!}{(n-r)!\,r!} $$

$$ {}_nC_2 = 630 $$

$$ 630 = \frac{n!}{(n-2)!\,2!} $$

$$ 630 = \frac{n(n-1)\cancel{(n-2)(n-3)}}{\cancel{(n-2)!}\,2!} $$

$$ \frac{630}{2} = \frac{n(n-1)}{2} $$

$$ 315 = n(n-1) $$

$$ 315 = n^2 - n $$

$$ n^2 - n - 315 = 0 $$

$$ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} $$

$$ x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-315)}}{2(1)} $$

$$ x = \frac{1 \pm \sqrt{1261}}{2} $$

$$ \swarrow \qquad \searrow $$

$$ \boxed{x = 18.248} \qquad \cancel{-17}.25 $$

24

extraneous

Use the following information to answer the next part of the written-response question.

A student is asked to encode the word **FACTOR** by replacing each letter of the word with a different math symbol. There are 10 math symbols available. The student creates a key, a list of his letter-symbol replacements, and gives it to his partner to use to decode the word.

b. **Explain** how you would determine the number of different keys that can be created for the word **FACTOR**, and **determine** the number of possible keys using your strategy. **[2 marks]**

10 Symbols

Factor = 6 letters

explination) there is 6 letters in the word factor and for each letter there is 10 math Symboles. ∴ to get the number of keys I will start with 10 choices mulipled by 1 less each time becaus 1 math Symbol can not be two letters. this will stop after 6 letters are Completed

Solving)

$$\underline{10} \times \underline{9} \times \underline{8} \times \underline{7} \times \underline{6} \times \underline{5} = 151,200$$

or

$$\frac{10!}{4!} = 151,200$$

Total Score - 2.5 marks	Rationale
Part a: 1 mark **Part b: 1.5 marks**	In part a, the response includes valid initial algebraic steps but does not determine the correct resulting quadratic equation. In part b, the response includes a partial explanation and the correct number of keys. The explanation does not explicitly address the fact that the order of the symbols is important.